# Modeling of sharp change in magnetic hysteresis behavior of electrical steel at small plastic deformation

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In 2.2% Si electrical steel, the magnetic hysteresis behavior is sharply sheared by a rather small plastic deformation (0.5%). A modification to the Jiles–Atherton hysteresis model makes it possible to model magnetic effects of plastic deformation. In this paper, with this model, it is shown how a narrow hysteresis with an almost steplike hysteresis curve for an undeformed specimen is sharply sheared by plastic deformation. Computed coercivity and hysteresis loss show a sharp step to higher values at small strain due to an $n=1/2$ power law dependence on residual strain. The step is seen experimentally. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1856191]

## I. INTRODUCTION

Investigations of the effect of plastic deformation on magnetic and x-ray data have been undertaken recently in 2.2% electrical steels.[1,2] The investigations produced the surprising result that even with a very small deformation $e_r = \Delta L/L = 0.005$, the magnetic hysteresis loop is strongly sheared.[2] This is despite the fact that in the undeformed steel, the hysteresis loop is very narrow and sharply steplike in appearance. X-ray data, reported here, appear to indicate that the dislocation density $\zeta_d$ is a linear function of the plastic deformation. On the other hand, direct measurement made of strain-hardening stress $\sigma_F$ vs residual plastic deformation $e_r$ indicates that $\sigma_F = Ce_r^n$, as in Ludwik's law,[3] but where $n \approx 1$. Since[4,5] dislocation density is proportional to $\sigma_F^2$, this means that a Ludwik-like law $\zeta_d = De_r^2$ exists, contrary to the x-ray data, and it also means that $H_c$, proportional to $\zeta_d^{1/2}$, should be linear in $e_r$, which again is not consistent with experimental data.

Recently, modifications to the Jiles–Atherton hysteresis model were made to take into account the effects of plastic deformation.[6] In this paper, we exhibit the first application of that model. We shall show that the model produces the sharp shearing of hysteresis curves, as in experimental data, and that it also produces steps in coercivity and hysteresis loss at small strain. This behavior is found by following the implications of x-ray data, which suggest that the Ludwik exponent should be $n=1/2$ and not $n=1$.

## II. EXPERIMENT

Several 2 m long, 0.5 mm thick electrical steel sheets were cold rolled along the original rolling direction, with different elongations. The steel had 2% Si, 0.4% Al, and 0.003% C. The yield stress was determined at 0.2% elongation, and coercive force was measured from maximum induction of 1.5 T, in quasistatic condition, at 90° from the rolling direction.[1,2]

Figure 8 in Ref. 2 shows hysteresis curves obtained for the undeformed, 0.5% deformed, and 5% deformed specimens. The deformed curves are sharply sheared compared with the undeformed curve. Coercive field is sharply increased at low deformations, as seen in Fig. 4 of Ref. 2. Sharp change in coercive field at low deformation can also be found in Hou and Lee[7] and in Hug *et al.*[8]

In our laboratory, we plotted $\sigma_F$ vs $e_r$ data on a log-log plot and obtained $n=1.057$ for the Ludwik exponent, finding, however, a lot of scatter in the data, particularly at low values of $e_r$. From this result, we would conclude that, for this sample, dislocation density approximately increases with the square of strain, which implies coercive force proportional to the strain. Our measurements, however, indicate that the coercive force increases less sharply, with a Ludwik exponent of $\approx 1/3$.

An additional problem is that the x-ray data, reported here, shows other behavior. Figure 1 shows (a) hysteresis loss $W_H$ vs $e_r$ and (b) x-ray peak width vs $e_r$. A sharp step appears in both sets of curves. A similar step is seen in $H_c$ vs

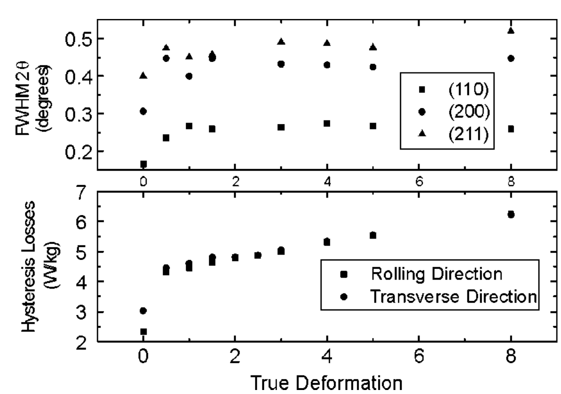


FIG. 1. Hysteresis loss $W_H$ vs $e_r$ and FWHM of x-ray peaks for different crystalline planes plotted against residual strain $e_r$. X-ray diffraction data obtained underts Cu $K_\alpha$ radiation, 40 kV, 20 mA.

 

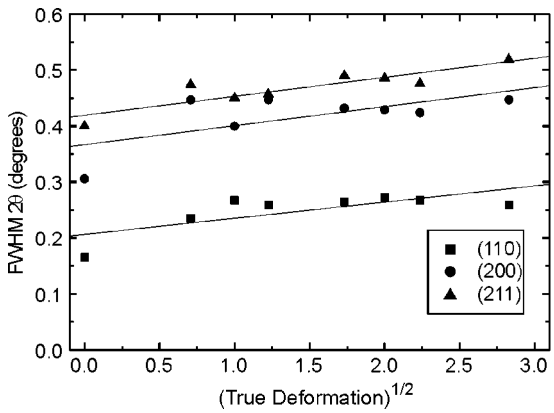


FIG. 2. Fit of the FWHM of the x-ray peak against $e_r^{1/2}$.

$e_r$. (See Fig. 4 in Ref. 2.) On the other hand, the x-ray peak width is also proportional to $e_r^{1/2}$, as in Fig. 2. Since[9] peak width is proportional to $\zeta_d^{1/2}$, it follows that $\zeta_d$ is proportional to $e_r$, which is not consistent with the Ludwik plot data giving $n=1.057$ (or $n\approx 1$), which implies $\zeta_d$ proportional to $e_r^2$.

With so many inconsistencies, it was hoped that hysteresis modeling would somehow elucidate everything. Indeed, it does help, as seen below. The modeling should be viewed as an effort to establish general trends, the details of which can be later resolved by taking nonuniform behavior into account.

## III. MODEL FORMULATION

The hysteresis model proposed for the effects of plastic deformation is discussed in Sablik *et al.*[6] It uses as its basic formulation that of Jiles and Atherton,[10] with effects of stress included as in Sablik and Jiles.[11] The modifications have to do with how the microstructure affects the parameters of Jiles and Atherton, namely, parameters $k$, $a$, $c$, $\alpha$, and $M_s$. In particular, the parameters $k$ and $a$ are both related to the coercivity,[12] are proportional to the square root of the dislocation density, and bear an inverse linear relationship with respect to the grain size, just as is the case with the coercivity.[12] While grain size is normally not affected by plastic deformation, the plastic deformation results in the formation of many new dislocations and in an increase of dislocation density as deformation continues. Thus, the chief effect of plastic deformation on magnetic properties is to increase coercivity and, through the effect of parameter $a$ scaling the effective magnetic field, to decrease the slope of the hysteresis loop.

We know from x-ray data that dislocation density is proportional to the residual plastic strain $e_r$. This implies that strain-hardening stress $\sigma_F$ is proportional to $e_r^{1/2}$ since the dislocation density increase is proportional to $\sigma_F^2$. Substituting $\sigma_F = A e_r^{1/2}$ into Astie–Degauque's equation[5,6]

$$\zeta_d = ([|\sigma_F|/(\alpha_K G b)] + \zeta_{d0}^{1/2})^2, \tag{1}$$

we obtain the dislocation density $\zeta_d$, where $\zeta_{d0}$ is the initial dislocation density in the undeformed state, $G$ is the shear modulus, $b$ is the Burgers vector magnitude, and $\alpha_K$ is 0.76. $\zeta_d$ is then substituted into expressions[6,12] for the Jiles–Atherton parameters $k$ and $a$ in order to determine the influence on magnetic hysteresis.

Another contribution to the modeling is in the stress term $H_\sigma$ in the effective field $H_e$ in the modified Sablik–Jiles model.[6] We had previously substituted that

$$H_\sigma = (3\sigma_r/2\mu_0) d\lambda/dM_a, \tag{2}$$

where $\sigma_r$ is the resulting residual stress after the specimen has been relaxed back to zero applied stress, $\lambda$ is the magnetostriction in the relaxed state, and $M_a$ is the anhysteretic magnetization in the relaxed state, defined as

$$M_a = M_s L(H_e/a), \tag{3}$$

where $M_s$ is saturation magnetization, $L(x)$ is the Langevin function written as $\coth(x) - 1/x$, $a$ is the material effective field scaling constant, and where $H_e$ is the effective magnetic field inside the material, given here as

$$H_e = H + \alpha M + H_\sigma, \tag{4}$$

We use this formulation, but reinterpret what to write for residual stress. Rolling stress is applied, which means that compressive stress is applied normally to the surface. However, the magnetic properties were measured along directions in the plane of the specimen surface, so the magnetic effects are responsive to the stress found transverse to the specimen normal. In the elastic regime, following plastic deformation, the transverse strain $e_t$ is tensile and has magnitude $-\nu e_c$, where $\nu$ is Poisson's ratio and $e_c$ is the normal compressive strain, which is negative, yielding positive transverse tensile strain. Because stresses and strains are linearly related in the elastic regime, we also have that $\sigma_t = -\nu\sigma_c$. It is known that a similar relationship applies under plastic deformation,[13] in that parallel to the specimen surface, there is a predominant tensile strain. The factor $\nu$ is no longer constant, but rather a *function* of the plastic strain. As a first approximation, we write it as an average over plastic strain. We denote it as $\nu'$, the normal stress factor. With $\sigma_f$ as the final applied stress at largest deformation and $\sigma_y$ as the yield stress, the part of the stress producing the slip is the strain hardening stress $\sigma_F = \sigma_f - \sigma_y$. It is this stress that we write as the residual stress, and in the normal direction. The transverse residual stress is thus $\sigma tr = -\nu'\sigma_F$, and so

$$H_\sigma = -(3\nu'\sigma_F/2\mu_0) d\lambda/dM_a. \tag{5}$$

The reader is asked to refer to Sablik *et al.*[6] for the rest of the model formulation. Here, because of using the Ludwik relation for the strain-hardening curve, the construction[6] using fraction $f$ is no longer necessary.

## IV. MODEL RESULTS

Figure 3 shows the results of the computation using the new model formulation. For this, the Jiles–Atherton parameters are $M_s = 1.212\times10^6$ A/m, $c = 6.137\times10^{-4}$, and $\alpha = 1.270\times10^{-4}$, with $k_0$ and $a_0$ for grain size 20 $\mu$m and dislocation density $1\times10^{12}/\text{m}^2$ given by $k_0/\mu_0 = 34.61$ A/m and $a_0 = 56.82$ A/m. The saturation magnetostriction is given by $\lambda_s = 7.14\times10^{-6}$; the constants $G_1 = 0.8100\times10^{-6}$ m and $G_2 = 3.800\times10^{-12}$ m$^2$; Young's modulus is given by 1.932

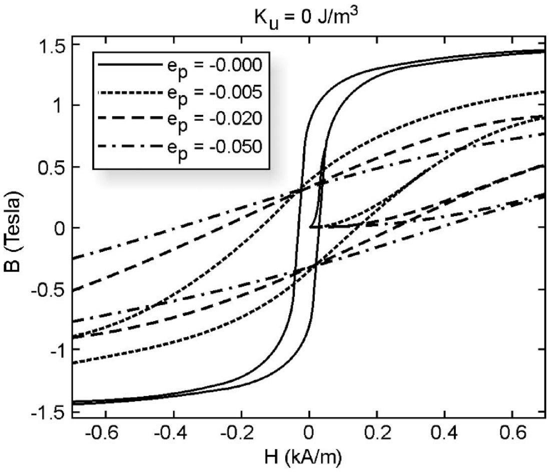


FIG. 3. Computed hysteresis curves using Ludwik's equation for $\sigma_F$ vs $e_r$ with $n=1/2$ and using the new expression for residual stress $\sigma_r$. Hysteresis curves are shown for zero deformation and for deformations of 0.005, 0.02, and 0.05. The loops at nonzero deformations are not shown in full, as they are cut off at $H_{max}=700$ A/m.

$\times 10^{11}$ J/m$^3$. It is known that $\nu'$ can be as large as 0.5 under plastic deformation,[13] but here under small deformation, we choose a value[9] close to the value it would have under elastic deformation. Thus, $\nu'$ is taken as 0.33, although it should be noted that a larger normal stress factor $\nu'$ (close to 0.5) has only a secondary effect and does not strongly influence the hysteresis. Finally, hysteresis curves are all taken to $B_{max}$ $=1.5$ T. The Ludwik constant $C$ for relating the strain-hardening stress $\sigma_F$ to the residual plastic strain $e_r$ is taken as $C=2.5\times 10^{18}$ J/m$^3$.

It is seen clearly in Fig. 3 that the zero deformation hysteresis curve is quite narrow and has a sharp step in it. The hysteresis curves for $e_r=0.005$, 0.02, and 0.05 are all progressively sheared strongly, with slopes decreasing with increasing deformation $e_r$. Although the strong shearing appears to be a byproduct of the initial steplike shape of the undeformed curve, details of the exact slopes and of the dependence of $H_c$ and $W_H$ on $e_r$ will be determined by the Ludwik exponent that is eventually found to fit the data. The fact that the Ludwik exponent is less than 1 has the effect of

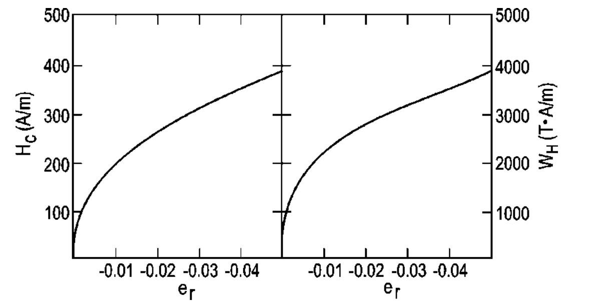


FIG. 4. Computed plots of $H_c$ vs $e_r$ and $W_H$ vs $e_r$. In this plot, it is seen that a Ludwik exponent $n=\frac{1}{2}$ results in both $H_c$ and $W_H$ having an $e_r^{1/2}$ dependence.

increasing the shearing of the loops. Another result is that $H_c$ is proportional to the square root of the residual strain. The result is seen in Fig. 4, where both coercivity $H_c$ and hysteresis loss $W_H$ exhibit an initial sharp step in the dependence on residual strain $e_r$, just as in experimental data, seen for $W_H$ in Fig. 1. The $e_r^{1/2}$ dependence of $H_c$ is somewhat consistent with what was found experimentally, which showed $H_c$ proportional to the 1/3 power of $e_r$.

## ACKNOWLEDGMENTS

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